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Primary school teachers' beliefs and knowledge about mathematical problem-solving and their performance in a problem-solving task

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ABSTRACT: This article describes primary teachers' beliefs, knowledge and performance regarding mathematical problem-solving. Explorative descriptive research was undertaken involving 80 teachers from East Java Province, Indonesia. Data were obtained through questionnaires and problem-solving tasks. The results of this study indicate that the teachers have a sufficient understanding of the knowledge of problem-solving as instruction and problem-solving in teaching practice. However, they have less understanding about the knowledge of problem-solving strategies and the meaning of mathematical problems. It can be explained from the teachers' performance in problem-solving tasks, indicating that their incorrect answers were found to be a manifestation of their difficulties in applying problem-solving strategies. Analysis of teachers' beliefs show that the teachers tend to view mathematics as an instrumental tool, while they tend to view teaching mathematics as a learner focussed task and students should learn mathematics as an autonomous exploration of students' own interest, which is aligned with a problem-solving view.

INTRODUCTION

It is widely accepted that teachers' beliefs and knowledge play an important role in the planning and implementation of teachers' teaching [1-3]. Andrews and Hatch [4] argued that beliefs influence what a teacher will teach, how it is taught and, therefore, what is learned in the classroom. Additionally, Ernest [3] affirmed that belief and knowledge [5] also affect student learning outcomes and a change in beliefs is considered to be a prerequisite for changes in teaching practice. Teacher knowledge, on the other hand, is important for identifying students' mathematical problem-solving proficiency within practical teaching. For example, being able to recognise the possible students' strategies in problem-solving allows teachers to interpret why a particular problem could be difficult. Moreover, being able to choose a suitable problem, and understand the nature of it, is also an important part of a problem-solving lesson [6]. This knowledge, as Franke and Kazemi stated, could help teachers to understand which characteristics make problems difficult for students and why [7].

As per Ball et al, general mathematical ability does not fully account for the knowledge and skills needed for effective mathematics teaching [8]. Teachers, they said, need a special type of knowledge to teach effectively problem-solving, which should be more than general problem-solving ability. Thus, Chapman has formulated a set of problem-solving knowledge needed by teachers regarding problem-solving, two of which are problem-solving content knowledge and problem-solving pedagogical knowledge [9]. Knowledge of problem-solving contains the knowledge of the meaning of problems, open-ended problems, problem-solving steps, problem-solving strategies, implementation of problem-solving in teaching practice.

The authors' previous study on secondary teachers found that teachers have sufficient understanding of pedagogical problem-solving knowledge despite it being indicated that they have less knowledge of problem-solving content knowledge, such as problem-solving strategies and the meaning of the problem itself [10]. Regarding beliefs, they tend to view both mathematics and how students should learn mathematics as a Platonist view, while they tend to believe in applying the idea of problem-solving as a dynamic approach when teaching mathematics. In this present study, the authors tried to undertake an investigation with a different participant focus that is primary teachers. Hence, drawing from the work of Beswick's [1] in a summary about mathematics-related beliefs and Chapman's [9] category of problem-solving knowledge for teaching, the authors described primary teachers' knowledge and beliefs about problem-solving, as well as their performance in problem-solving tasks as the manifestation of their proficiency regarding mathematical problem-solving.

LITERATURE REVIEW

Beliefs in mathematical problem-solving are closely related to beliefs about the nature of mathematics, as well as teaching and learning mathematics. Viholainen et al explained that beliefs about the nature of mathematics influence beliefs concerning mathematical problem-solving or *vice versa*, and those beliefs concerning the learning of mathematics also imply beliefs about the teaching of mathematics [11]. Meanwhile, Ernest stated that teachers' view of the nature of mathematics affects how they play their role in classroom teaching and learning [3]. To that, he presents three different philosophical views of the nature of mathematics: instrumental, Platonist and problem-solving. In attempt to simplify these views, Beswick summarised connections among the nature of mathematics, mathematics learning and mathematics teaching (see Table 1) [1].

Table 1: Summary of beliefs about mathematics, mathematics teaching and mathematics learning.

Beliefs about the nature of mathematics	Beliefs about mathematics teaching	Beliefs about mathematics learning
Instrumentalist	Content focussed with an emphasis on performance	Skill mastery, passive reception of knowledge
Platonist	Content focussed with an emphasis on understanding	Active construction of understanding
Problem-solving	Learner focused	Autonomous exploration of own interests

Table 1 explains that despite a theoretical consistency between each of the corresponding beliefs in the same row shown in Table 1, it does not guarantee that it holds consistency among the beliefs of individual teachers. For instance, it is possible that a Platonist teacher shows beliefs about teaching mathematics, which emphasise student performance rather than student understanding. Regarding teacher knowledge, Table 2 shows Chapman's category of problem-solving knowledge for teaching in detail [9].

Table 2: Knowledge needed in understanding problem-solving.

Type of knowledge	Knowledge	Description
Problem-solving content knowledge	Mathematical problem-solving proficiency	Understanding what is needed for successful mathematical problem-solving.
	Mathematical problems	Understanding of the nature of meaningful problems; structure and purpose of different types of problems; impact of problem characteristics on learners.
	Mathematical problem-solving	Being proficient in problem-solving. Understanding of mathematical problem-solving as a way of thinking; problem-solving models and the meaning and use of heuristics; how to interpret students' unusual solutions; and implications of students' different approaches.
	Problem posing	Understanding of problem posing before, during and after problem-solving.
Pedagogical problem-solving knowledge	Students as mathematical problem solvers	Understanding what a student knows, can do and is disposed to do.
	Instructional practices for problem-solving	Understanding how and what it means to help students to become better problem solvers.
Affective factors and beliefs		Understanding nature and impact of productive and unproductive affective factors and beliefs on learning and teaching problem-solving and teaching.

Table 2 shows a number of subtypes of knowledge of each problem-solving content knowledge and problem-solving pedagogical knowledge. In particular, the specific issue related to problem-solving content knowledge, which is being proficient in problem-solving as one of the indicators of knowledge of mathematical problem-solving, becomes another interest in this study. Thus, to examine this specific knowledge, the authors are interested in obtaining information about teachers' performance in problem-solving tasks. It appears critical that teachers should also be proficient to deal with a variety of problem-solving tasks, such as completing problem-solving steps, as well as applying problem-solving strategies to various mathematical tasks.

METHOD

This is descriptive explorative research involving 80 primary teachers who have a minimum of a Bachelor degree, have taught for more than five years, from Surabaya, Sidoarjo, Gresik and Mojokerto (East Java). Data were collected from questionnaire and problem-solving tasks. The questionnaire consisted of 18 multiple choices questions. Each item provided four to 17 choices. Some of those questions had a large number of choices, because of the need to cover as many teacher's responses as possible, both correct and incorrect. For instance, the question item: *A mathematical question is called a mathematical problem when...* had six choices consisting of three correct answers and three incorrect answers. Thus, the teacher could make more than one choice. To explore the teachers' knowledge about problem-solving, there were 15 items categorised into seven groups of questions. The categorisation of these groups was based on Chapman's type of problem-solving knowledge described in Table 2. The groups are: a) problem-solving content knowledge: meaning of the problem (one item), open-ended problem (one item), problem-solving as instruction (one item), problem-solving steps (three items), problem-solving strategies (two items); and b) pedagogical problem-solving knowledge: instructional practice of problem-solving (three items) and designing problem-solving task (three items). As an example, the authors give one question including its options as follows:

An elementary student shows how to sum 1+2+3+4+5+6+7+8+9+10+11+12+13+14+15 by firstly drawing a table as follows:

Pattern	The pattern is obtained from...	Note
1+2 = 3	3 is obtained from $\frac{2x3}{2} = 3$	2 is the last number and continued with 3 as the subsequent number
1+2+3 = 6	6 is obtained from $\frac{3x4}{2} = 6$	3 is the last number and continued with 4 as the subsequent number
1+2+3+4 = 10	10 is obtained from $\frac{4x5}{2} = 10$	4 is the last number and continued with 5 as the subsequent number

From the table, the student conclude that the sum of the series is 1+2+3+….+15= $\frac{15 \text{ x } 16}{2}$ = 120

The strategy most likely used by the student is by:

a) Applying the formula of the sum of arithmetical series;
b) Guessing and checking in trial and error;
c) Considering a simple pattern of the sum of smaller number of terms, and applying the pattern to the bigger number of terms;
d) Considering all possibilities of answer;
e) Drawing a sketch or picture representing the problem-solving process.

A descriptive analysis of teachers' performance was carried out by classifying the teachers' responses on the problem-solving tasks into correct and incorrect answers by considering a variety of types found in the teachers' responses. Meanwhile, the teachers' beliefs were established by using percentages of the number of options selected by the teachers categorised into three philosophical beliefs: instrumentalist, Platonist and problem-solving.

RESULTS

Teachers' Knowledge about Mathematical Problem-solving

Table 3: Teachers' knowledge about mathematical problem-solving.

Knowledge	Category	Number of options selected by teacher	Percentage
Meaning of problem	Incorrect	66	61.11%
	Correct	42	38.89%
Open-ended problem	Incorrect	53	44.92%
	Correct	65	55.08%
Problem-solving as instruction	Incorrect	9	9.09%
	Correct	90	90.91%
Problem-solving stages	Incorrect	188	42.25%
	Correct	257	57.75%
Types of problem-solving strategies	Incorrect	198	74.44%
	Correct	68	25.56%
Implementation of steps and strategies of problem-solving in teaching	Incorrect	151	49.51%
	Correct	154	50.49%
Posing problem-solving task	Incorrect	131	38.53%
	Correct	209	61.47%

Table 3 shows that many teachers did not understand the meaning of *problem*. There were 61.11% incorrect options chosen by teachers. Teachers also had less understanding about types of problem-solving strategies, i.e. 74.44% did not understand such knowledge. However, they had sufficient knowledge about open-ended problems, problem-solving as instruction, problem-solving steps, implementation and problem-solving experience when designing tasks. It shows that although teachers are aware of the importance of problem-solving as the focus of learning, there are still weaknesses in selecting a task question as a problem and solution strategies. These conditions are likely to cause weaknesses in teachers' ability to solve a problem. To confirm, teacher difficulties in solving problem-solving tasks is described as follows.

Teachers' Performance in Solving Problem-solving Tasks

Table 4: Teachers' performance in problem-solving tasks.

Problem	Type of responses	Total	Percentage of teachers	Problem	Type of responses	Total	Percentage of teachers
Problem 1	I	59	74.68%	Problem 2	I	17	21.52%
	II	14	17.72%		II	44	55.70%
	III	6	7.59%		III	16	20.25%
	IV	0	0%		IV	1	1.27%
					V	1	1.27%

Table 4 shows that teachers' solutions to problems could be categorised into one of several types. For Problem 1, Type I (see the Appendix) is the correct answer. Type II is an incorrect answer since the teacher just compared the available data from given information without providing any sufficient manipulation of data. Type III is also an incorrect answer, because it is without mathematical arguments, such as the calculation of percentages. Type IV is that the teacher did not answer the question.

For Problem 2, Type I (see the Appendix) is a correct answer. Type II is incorrect since the teachers calculated each of the motorcycles that failed assembled by each company, but misinterpreted the conclusion. Type III is also an incorrect answer, because it simply sums the total failure of their respective companies. Type IV is also incorrect answer due to an incorrect calculation. Type V is that the teachers did not answer the problem.

Based on these data, the teachers are still experiencing difficulties in solving problems. For the first problem, 25% of teachers still answered incorrectly, while for the second problem, 78% of teachers were wrong.

Table 5 and Table 6 show some of the types for each of responses to Problem 1 and Problem 2, respectively.

Table 5: Examples of responses to Problem 1.

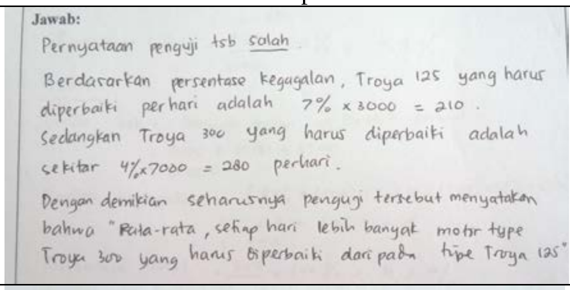
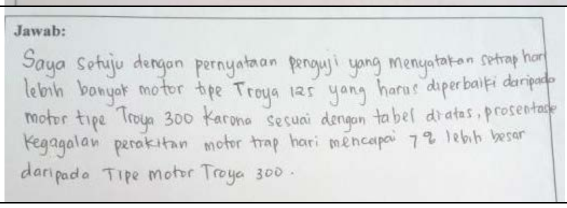
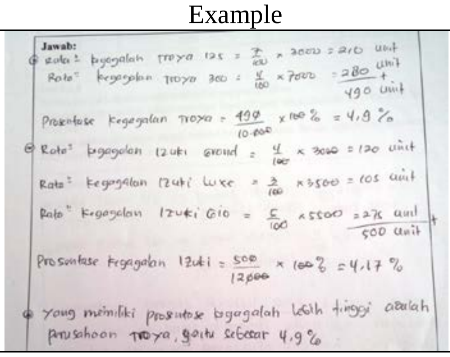
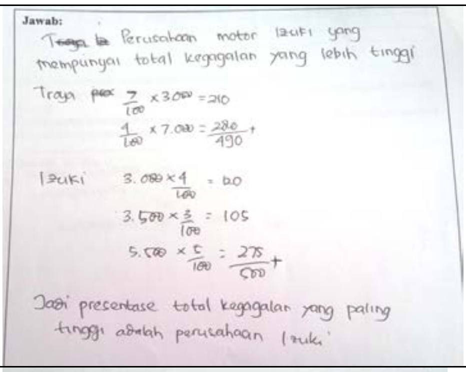
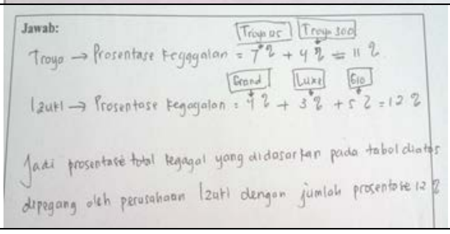
Type	Example	Translation
Type 1	 Jawab: Pernyataan penguji tsb salah Berdasarkan persentase kegagalan, Troya 125 yang harus diperbaiki per hari adalah 7% x 3000 = 210. Sedangkan Troya 300 yang harus diperbaiki adalah sekitar 4%x7000 = 280 perhari. Dengan demikian seharusnya penguji tersebut menyatakan bahwa "Rata-rata, setiap hari lebih banyak motor type Troya 300 yang harus diperbaiki daripada tipe Troya 125"	The statement is false, because the number of Troya 125 that should be repaired/day is 7% x 2000 = 210, while the number of Troya 300 that should be repaired is 4% x 700 = 280.
Type 2	 Jawab: Saya setuju dengan pernyataan penguji yang menyatakan setiap hari lebih banyak motor tipe Troya 125 yang harus diperbaiki daripada motor tipe Troya 300 karena sesuai dengan tabel diatas, prosentase kegagalan perakitan motor tiap hari mencapai 7% lebih besar daripada Tipe motor Troya 300.	I agree with the statement that there are more Troya 125 should be repaired than Troya 300 as presented in the table.

Table 6: Examples of responses to Problem 2.

Type	Example	Translation
Type 1	 Jawab: Rata² kegagalan Troya 125 = 7/100 x 3000 = 210 unit Rata² kegagalan Troya 300 = 4/100 x 7000 = 280 unit + 490 unit Prosentase kegagalan Troya = 490/10.000 x 100% = 4,9% Rata² kegagalan Izuki Grand = 4/100 x 3000 = 120 unit Rata² kegagalan Izuki Luxe = 3/100 x 3500 = 105 unit Rata² kegagalan Izuki Gio = 5/100 x 5500 = 275 unit + 500 unit Prosentase kegagalan Izuki = 500/12000 x 100% = 4,17% Yang memiliki prosentase kegagalan lebih tinggi adalah perusahaan Troya, yaitu sebesar 4,9%	Average faulty number of Troya 125 is 7% x 3000 = 210 Average faulty number of Troya 300 is 4% x 7000 = 280 Percentage of faulty number of Troya is 490/10000 = 4.9% Average faulty number of Izuki Grand is 4% x 3000 = 120 Average faulty number of Izuki Luxe is 3% x 3500 = 105 Average faulty number of Izuki Gio is 5% x 5500 = 275 Percentage of faulty number of Troya is 500/12000 = 4.9%

Type 2		Izuki has a higher total percentage of faultily assembled motor: Troya $\frac{7}{100} \times 3000 = 210$ $\frac{7}{100} \times 7000 = 280$ Added becomes 490 Troya $\frac{4}{100} \times 3000 = 120$ $\frac{7}{100} \times 3500 = 105$ $\frac{7}{100} \times 5500 = 275$ Added becomes 500
Type 3		Troya → percentage of faulty = 7% + 4% = 11% Izuki → percentage of faulty = 4% + 3% + 5% = 12% So, the total percentage of faultily assembled motorcycle is 12%

Teachers' Beliefs about Mathematical Problem-solving

Table 7: Teachers' beliefs about mathematical problem-solving.

Beliefs	Philosophical category	Number of options selected by teachers	Percentage
What is mathematics?	Instrumental	38	24.05%
	Platonist	27	17.09%
	Problem-solving	68	21.52%
How to teach mathematics?	Content focussed with an emphasis on performance	41	12.97%
	Content focussed with an emphasis on understanding	49	15.51%
	Learner focussed	130	18.28%
How should students learn mathematics?	Skill mastery, passive reception of knowledge	72	18.23%
	Active construction of understanding	31	13.08%
	Autonomous exploration of own interests	83	26.27%

Table 7 shows that teachers still tend to view mathematics as a tool (24.05%), which is higher than the view of mathematics in a Platonist view (17.09%) and mathematics in a problem-solving view (21.52%). However, in viewing teaching and learning mathematics, the primary teachers are more likely to believe that mathematics is aligned with the problem-solving view, i.e. 18.28% and 26.27%, respectively. This fact shows that the belief in mathematics is not the only factor affecting the practice of teaching and the views of the students who are learning. Raymond describes other factors besides belief in mathematics, such as teacher education programmes, social teaching norms, teachers' life outside the classroom, characteristics of teacher's personality, the situation in the classroom and student life outside the classroom [12].

CONCLUSIONS

To conclude, the authors highlight two results regarding teachers' knowledge and beliefs. With regards to teachers' knowledge, teachers who understand the meaning of problems and the strategies of problem-solving, were less represented than those who understand the implementation of problem-solving in teaching practice. Such a lesser amount of knowledge can be explained by teachers' performance in the problem-solving task indicating that their incorrect answers were found to be the manifestation of their difficulties in applying problem-solving strategies and interpreting the results of their work back to the initial problem. However, teachers who understand the characteristics of open-ended problems, teaching with problem-solving steps including its implementation were more frequent than those who do not understand.

With regard to teachers' beliefs, teachers believed more in mathematics as an instrumental view. However, in accordance with viewing on how to teach mathematics and how students learn mathematics, more teachers believed in those two types of beliefs as learner focussed teaching and an autonomous exploration of students' own interest, which is aligned with a problem-solving view.

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APPENDIX

The Troya brand motorcycle company assembles two of the latest types of motorcycle: Troya 125 and Troya 300. On the other hand, another motorcycle company, Izuki, assembles three types of the latest motorcycles: Grand, Luxe and Gio. The following table shows the comparison between the number of motorcycles assembled and the percentage of faultily assembled motorcycles for both the two companies.

Company	Type of motorcycles	Average number of assembled motorcycles per day	Average percentage of faultily assembled motorcycles per day
Troya	Troya 125	3,000	7%
	Troya 300	7,000	4%
Izuki	Grand	3,000	4%
	Luxe	3,500	3%
	Gio	5,500	5%

Problem 1

An examiner gives a statement, …*on average, there are more Troya 125 motorcycles that need to be repaired than those of Troya 300 per day*. Is the statement true? Explain your argumentation.

Problem 2

Which of the two companies, Troya or Izuki has the higher overall percentage of faultily assembled motorcycles? Show your work using data in the table above.